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TENSOR PRODUCT AND l-TWO PARANORMED STRUCTURES

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ABSTRACT : The paper generalizes the idea of tensor product of two normed linear spaces to that of two l-paranormed spaces relative to a given l-paranormed space and discusses different types of l-paranorms which generate different types of l-two paranorm structures on the tensor product space. It also considers the tensor product of a Banach algebra and a complete l-paranormed algebra relative to a given complete l-paranormed algebra and shows that, in this case, the tensor product is a l-two paranormed algebra.

Keywords : Tensor Product, l-two Paranormed space, l-two Paranormed algebra.

1. INTRODUCTION : The distinction between a homogeneous norm and a nonhomogenous norm was made by W. Orlicz (1950); he called them a B-norm and a F-norm respectively. Later on, B. Chaudhury and S. Nanda (1991) called such a nonhomogeneous norm a paranorm. A. Taylor (1958) defined a metric linear space and L. Collatz (1966) defined a supermetric space. Moreover, they showed that every metric linear space or a supermetric space (X, ρ) was such that ρ could induce such paranorm p on the linear space x given by $p(x) = \rho(x, \theta)$,θ being the additive identity of X. D. K. Bhattacharya and T. Roy (1997) defined a l-paranormed space and a l-paranormed algebra and studied some properties of such an algebra. They also showed that every l-paranormed space was a paranormed space but the converse was not true. Also, it followed from the definitions that a normed linear space was not a l-paranorm space. As normed linear spaces were obviously paranormed spaces, so it was remarked that normed linear spaces and l-paranormed spaces formed two distinct subsets of the set all paranormed spaces. This necessitated further study of such l-paranormed spaces and algebras.

The idea of a two normed space was introduced by W. Orlicz (1950) and the importance of such a space was subsequently highlighted by A. Alexiewicz and Z. Semadeni [1954, 1958, 1960] and several others.

The idea of a two normed algebra was given by R. K. Srivastava (1990) while studying the space of entire Dirichlet series. D. Bhattacharya and S. Manna (1997) defined a two paranormed space and different types of two paranormed algebras and cited examples from the set of entire Dirichlet series with different exponents.

Extensive work was done by R. Shattern (1950), A. Grothendieck (1955), B. Gelbaum (1959), F. Bonsall and J. Duncan (1973) and many others on the tensor product of two Banach spaces and on that of two Banach algebras. In the latter case, out of the three possible types of norms on such a space, only one norm viz. projective norm was an algebra norm. As in the definition of a two normed algebra, both types of norms were to be algebra norms, so it remained open to investigate whether the tensor product of two Banach algebras was a two normed algebra.

In the present paper, we answer the question in the affirmative by considering the tensor product of a Banach algebra and a complete l-paranormed algebra relative to another complete l-paranormed algebra.

Throughout the paper, we use the definitions and examples as given in [4].

2.SOME NEW DEFINITIONS AND EXAMPLES:

DEFINITION 2.1. Let X be a linear space equipped with two l-paranorms p and p^* where p^* is weaker than p. A sequence $\{x_n\} \subset X$ is said to be γ-convergent to $x_0 \in X$, if there exists k > 0 such that $\sup_n p(x_n) \le k < \infty$ and $p^*(x_n - x_0) \to 0$ as $n \to \infty$.

DEFINITION 2.2. A linear space x equipped with two l-paranorms p and p^* (p^* being weaker than p) is called a l-two paranormed space if convergence of a sequence of x means its γ-convergence. It is denoted by $(x, p, \cdot p^*)$ and is abbreviated as l-TPS.

DEFINITION 2.3. A sequence $\{x_n\}$ of a l-TPS (x, p, p^*) is called a Cauchy sequence if there exists k > 0 such that $\sup_n p(x_n) \leq k < \infty$ and if corresponding to $\varepsilon > 0$, arbitrary small, there exists a positive integer m such that $p^*(x_p - x_q) < \varepsilon$, for $\forall$ p, $q \geq m$.

A l-TPS is called ϑ-complete if every Cauchy sequence is ϑ-convergent in the space.

EXAMPLE 2.1. The class of all real sequences x = $\{x_n\}$ with componentwise addition and scalar multiplication is a l-TPS if p and p^* are defined as p(x) = $\sum_{n=1}^{\infty} \frac{|x_n|}{1+|x_n|}$ and $p^*(x) = \sum_{n=1}^{\infty} \frac{1}{2^n} \frac{|x_n|}{(1+|x_n|)}$. Other examples of l-TPS are considered in the next two articles.

3. BOUNDED LINEAR MAPS AND l-TWO PARANORM SPACES

DEFINITION 3.1. Let (X, p_x), (Y, p_y) be two l-paranormed spaces over the field F. A linear map $T : X \to Y$ is said to be bounded if $p_y\left(T\left(\frac{x}{p_x(x)}\right)\right) \leq k$, $\forall\, x \in X,\ x \neq \theta$ (null element of X)

PROPOSITION 3.1. Let B(X, Y) denote the set of all bounded linear maps

$T : (X, p_x) \to (Y, p_y)$, Let

(a) p(T) = inf k such that $p_y\left(T\left(\frac{x}{p_x(x)}\right)\right) \leq k$, $\forall\, x \in X,\ x \neq \theta$.

(b) $p(T) = \sup_x \left\{ p_y\left(T\left(\frac{x}{p_x(x)}\right)\right),\ x \in X,\ x \neq \theta \right\}$

Then in both the cases p is a l-paranorm on B(X,Y). Moreover these two p's are same.

PROOF. In order to show that (a) and (b) give l-paranorms of T, we simply verify that in both the cases.

$p(\alpha T) \leq |\alpha|\, p(T)$, $|\alpha| \geq 1$

$|\alpha|\, p(T) < p(\alpha T) < p(T)$, $|\alpha| < 1$, $\alpha \in F$

We have,

$$p_y\left(\alpha\, T\left(\frac{x}{p_x(x)}\right)\right) < |\alpha|\; p_y\left(T\left(\frac{x}{p_x(x)}\right)\right), \quad x \neq \theta \text{ , if } |\alpha| > 1$$

$$< |\alpha|\; k \text{ where } p_y\left(T\left(\frac{x}{p_x(x)}\right)\right) \leq k\,,\ \forall\, x \neq \theta.$$

$$\text{So } \ p_y\left(\alpha\, T\left(\frac{x}{p_x(x)}\right)\right) < |\,\alpha'\,|\, k \text{ for some suitable } |\,\alpha'\,| < |\alpha|.$$

and hence for $|\alpha| > 1$, $p(\alpha\, T) = \inf |\,\alpha'\,|\, k = |\,\alpha'\,| \inf k$

$$= |\,\alpha'\,|\, p(T) < |\alpha|\, p(T)$$

Again for $|\alpha| = 1$, $p(\alpha\, T) = p(T)$.

So, $p(\alpha\, T) \leq |\alpha|\, p(T)$, $|\alpha| \geq 1$.

$$\text{Again } |\alpha|\; p_y\left(T\left(\frac{x}{p_x(x)}\right)\right) < p_y\left(\alpha\, T\left(\frac{x}{p_x(x)}\right)\right) < p_y\left(T\left(\frac{x}{p_x(x)}\right)\right), \quad |\alpha| < 1\,,\ \forall\, x \neq \theta.$$

$$\text{So, } p_y\left(\alpha\, T\left(\frac{x}{p_x(x)}\right)\right) \leq k' < k\,, \text{ if } p_y\left(T\left(\frac{x}{p_x(x)}\right)\right) \leq k\,,\ \forall\, x \neq \theta.$$

Hence, $p(\alpha\, T) = \inf k' < \inf k = p(T)$ $(\text{when } |\alpha| < 1)$

Thus when $|\alpha| < 1$, $p(\alpha\, T) < p(T)$.

$$\text{Again } |\alpha|\; p_y\left(T\left(\frac{x}{p_x(x)}\right)\right) < p_y\left(\alpha\, T\left(\frac{x}{p_x(x)}\right)\right)$$

$$\Rightarrow |\alpha|\; p_y\left(T\left(\frac{x}{p_x(x)}\right)\right) \leq \inf k' \text{ such that } p_y\left(\alpha\, T\left(\frac{x}{p_x(x)}\right)\right) \leq k'$$

$$\Rightarrow |\alpha|\; p_y\left(T\left(\frac{x}{p_x(x)}\right)\right) \leq p(\alpha\, T)$$

$$\Rightarrow p_y\left(T\left(\frac{x}{p_x(x)}\right)\right) \leq \frac{1}{|\alpha|}\, p(\alpha\, T)\,,\ \forall\, x \neq \theta\ (\alpha \neq 0)$$

$$\Rightarrow p(T) = \inf\left(\frac{1}{|\alpha|}\, p(\alpha\, T)\right) < \frac{1}{|\alpha|}\, p(\alpha\, T)$$

i.e, $|\alpha|\, p(T) < p(\alpha\, T) < p(T)$ when $|\alpha| < 1$.

So, p is a l-paranorm defined by (a). Further, for the case (b) we have,

$$p(\alpha T) = \sup_x p_y\left[\alpha T\left(\left(\frac{x}{p_x(x)}\right)\right)\right] \leq |\alpha| \sup_x p_y\left(T\left(\frac{x}{p_x(x)}\right)\right) \text{ if } |\alpha| \geq 1$$

$$\text{Also } |\alpha| \sup_x p_y\left[T\left(\frac{x}{p_x(x)}\right)\right] < \sup_x p_y\left[\alpha T\left(\frac{x}{p_x(x)}\right)\right] < \sup_x p_y\left(T\left(\frac{x}{p_x(x)}\right)\right) \text{ if } |\alpha| < 1$$

So $p(\alpha T) \leq |\alpha|\, p(T)$ if $|\alpha| \geq 1$.

$|\alpha|\, p(T) < p(\alpha T) < p(T)$ if $|\alpha| < 1$.

Thus p is a l-paranorm for the case (b) also.

To show that the above two l-paranorms are same,

let us denote the right hand sides of (a) and (b) by M_1 and M_2.

Obviously, $M_2 \leq M_1$. Again, from (b) it follows that

$p_y\left(T\left(\frac{x}{p_x(x)}\right)\right) \leq M_2$, so that from (a) we have $M_2 \geq M_1$.

Hence $M_1 = M_2 = p(T)$.

PROPOSITION 3.2. $p^*(T) = \sup\{p_y(T(x))\,;\, p_x(x) \leq 1\}$ is a l-paranorm on B(X, Y) and $p^*(T) \leq p(T)$, $\forall\, T \in B(X, Y)$

PROOF : $p^*(T)$ is obviously a l-paranorm on B (X, Y). Now we show that $p^*(T) \leq p(T)$.

First we see that $S = \{x \in X \,/\, p_x(x) \leq 1\}$

$$\subset S_1 = \left\{\{0\} \cup \left\{\frac{x}{p_x(x)}\right\},\ x \neq 0,\ x \in X\right\}$$

In fact, for those x which satisfy $p_x(x) \leq 1$,

$$p_x\left(\frac{x}{p_x(x)}\right) \leq \frac{1}{p_x(x)} \cdot p_x(x) = 1,\ x \neq \theta$$

But all $x' \in X$, $x' = \dfrac{x}{p_x(x)}$, $x \in X$, $x \neq \theta$ may not satisfy $p_x(x') \leq 1$;

In fact, for these $x' = \dfrac{x}{p_x(x)}$ for which $p_x(x) > 1$,

$$1 < p_x(x') = p_x\left(\frac{x}{p_x(x)}\right) < p_x(x).$$

This implies that $p_x(x')$ can never be less than or equal to 1.

So $p^*(T) \leq p(T)$ where $p(T)$ is a l-paranorm given by (b).

PROPOSITION 3.3. Let B be the set of all bounded linear transformations

$T : (X , p_x) \rightarrow (Y , p_y)$ where (X , p_x) and (Y , p_y) are both l-paranormed spaces.

Let $p^*(T) = \sup_x \{p_y(T(x)) , x \in X, p_x(x) \leq 1\}$ and

$$p(T) = \sup_x \left\{p_y\left(T\left(\frac{x}{p_x(x)}\right)\right), x \in X , x \neq \theta\right\}$$

Then (B, p, p^*) is a l-two paranormed space.

4. BOUNDED LINEAR MAPS AND l-TWO PARANORMED SPACES

DEFINITION 4.1. Let (X, p_x), (Y, p_y), (Z, p_z) be three l-paranormed spaces. Let $\phi : X \times Y \rightarrow Z$ be a bilinear map, ϕ is said to be bounded if there exists

M > 0 such that $p_z\, \phi\left(\frac{x}{p_x(x)} , \frac{y}{p_y(y)}\right) \leq M$

$\forall x \in X , \forall y \in Y , x \neq \theta , y \neq \theta$.

The set of all such bounded bilinear maps is denoted by $\hat{B}(X, Y; Z)$

PROPOSITION 4.1.

Let X, Y, Z, ϕ be given as in definition 4.1.

Then $(\hat{B}, p, p^*)$ is a l-two paranormed space, where

$$p^*(\phi) = \sup_{x,\, y} \{p_z(\phi(x, y)) , p_x(x) \leq 1 , p_y(y) \leq 1\}$$

$$p(\phi) = \sup_{x,\, y} \left\{p_z\left(\phi\left(\frac{x}{p_x(x)} , \frac{y}{p_y(y)}\right)\right), x \neq \theta , y \neq \theta\right\}.$$

are two l-paranorms on $(\hat{B}, p, p^*)$.

5. TENSOR PRODUCT AND I-TWO PARANORMED ALGEBRA

5.1. TENSOR PRODUCT OF TWO I-PARANORMED SPACES W.R.T. ANOTHER I-PARANORMED SPACE

DEFINITION 5.1.

Let (X, p_x), (Y, p_y), (Z, p_z) be I-paranormed spaces over F.

Let X_z denote the set of all bounded linear maps from X to Z and let Y' denote the set of all bounded linear functionals on Y. Let $\hat{B}(X_z, Y' ; Z)$ denote the set of all bounded bilinear maps from $X_z \times Y' \to Z$. Given $x \in X$, $y \in Y$, let $x \otimes y$ denote an element of $\hat{B}(X_z, Y' ; Z)$ defined by $(x \otimes y)(f, g) = g(x) f(y)$ for all 1-1 maps $f \in X_z$ and $g \in Y'$. Then $(X \otimes Y)_z$, the algebraic tensor product relative to Z, is defined by the linear span of $\{x \otimes y , x \in x , y \in Y\}$ in $\hat{B}(X_z, Y' ; Z)$. When X and Y are both normed linear spaces over F, the usual algebraic tensor product of X and Y follows by taking Z = F.

The following propositions now easily follow :

PROPOSITION 5.1.

Given $u \in X \otimes Y$, there exists linearly independent sets $\{x_i\} \subset X$, $\{y_i\} \subset Y$, i = 1,2,3,.....n, such that $u = \sum_{i=1}^{n} x_i \otimes y_i$.

PROPOSITION 5.2.

If $u = \sum_{i=1}^{n} x_i \otimes y_i = 0$, where $\{ x_i \}$ is a linearly independent set, then $y_i = 0$; i = 1,2,....n.

PROPOSITION 5.3.

If $\{ x_i \}$; i = 1,2,....m and $\{ y_j \}$, j = 1,2,....n be two linearly independent subsets of X and Y, then $\{x_i \otimes y_i\}$ is a linearly independent subset of $X \otimes Y$.

The above propositions lead to the following

THEOREM 5.1. Let X, Y, Z be three I-paranormed spaces over F and let $\psi : (X \otimes Y)_z \to X \times Y$ be a bilinear map. Then corresponding to each bilinear map $\phi : X \times Y \to Z$, there exists a unique linear map $\sigma : (X \otimes Y)_z \to Z$ such that $\phi = \sigma \circ \psi$.

PROPOSITION 5.4.

Let X, Z be two complete I-paranormed algebras over F and let Y be a Banach algebra over F. Let X_z denote the set of all bounded linear maps from X to Z. Let

$u = \sum_{i=1}^{n} x_i \otimes y_i \in (X \otimes Y)_z$ and $w^*, p^* : (X \otimes Y) \to R$ be defined as.

$$w^*(u) = \sup_{f,g} \left(\sum_{i=1}^{n} \left| g(y_i) \right| p_z(f(x_i)) ; \| g \| \leq 1 , p_{x_z}(f) \leq 1 \right)$$

$$p^*(u) = \inf \left(\sum_{i=1}^{n} \| y_i \| \sup_f \left\{ p_z(f(x_i)) ; p_{x_z}(f) \leq 1 , f \in X_z \right\} \right)$$

where the infimum is taken over all finite representations of u.

Then w^* and p^* are both I-paranorms on $(X \otimes Y)_z$ and $w^*(u) \leq p^*(u)$, $\forall\, u \in (X \otimes Y)_z$.

PROOF : Obviously w^* and p^* are I-paranorms on $(X \otimes Y)_z$.

We simply verify that $w^*(u) \leq p^*(u)$.

Now $\sum_{i=1}^{n} \left| g(y_i) \right| p_z(f(x_i)) \leq \| g \| \sum_{i=1}^{n} \| y_i \| p_z(f(x_i))$

So, $\left(\sum_{i=1}^{n} \left| g(y_i) \right| p_z(f(x_i)) ; p_{x_z}(f) \leq 1 , \| g \| \leq 1 \right)$

$\leq \| g \| \left(\sum_{i=1}^{n} \| y_i \| p_z(f(x_i)) ; p_{x_z}(f) \leq 1 , \| g \| \leq 1 \right)$

$\leq \left(\sum_{i=1}^{n} \| y_i \| p_z(f(x_i)) ; p_{x_z}(f) \leq 1 \right)$

So, $w^*(u) = \sup_{f,g} \left\{ \sum_{i=1}^{n} \left| g(y_i) \right| p_z(f(x_i)) ; \| g \| \leq 1 , p_{x_z}(f) \leq 1 \right\}$

$\leq \left\{ \sum_{i=1}^{n} \| y_i \| \sup_f \left(p_z(f(x_i)) , p_{x_z}(f) \leq 1 \right) \right\}$

Hence $w^*(u) \leq \inf \left\{ \sum_{i=1}^{n} \| y_i \| \sup_f \left(p_z f(x_i) \right) ; p_{x_z}(f) \leq 1 \right\} = p^*(u)$

where the infimum is taken over all finite representations of u. Thus $w^*(u) \leq p^*(u)$.

PROPOSITION 5.5.

Let X, Y, Z, X_z be defined as in proposition 5.4.

$$\text{Let } w(u) = \sup_{f,\,g}\left(\sum_{i=1}^{n} \left| g(y_i) \right| p_z\left(\frac{f(x_i)}{p_z(f(x_i))}\right) ; \|g\| \leq 1,\ p_{x_z}(f) \leq 1,\ f \neq \theta_{x_z}\right)$$

$$\text{and } p(u) = \inf\left\{\sum_{i=1}^{n} \|y_i\| \sup_f \left(p_z\left(\frac{f(x_i)}{p_z(f(x_i))}\right)\right),\ p_{x_z}(f) \leq 1,\ f \neq \theta_{x_z}\right\}$$

where infimum is taken over all finite representations of u. Then w and p are both l-paranorms on $(X \otimes Y)_z$ such that

(i) $w(u) \leq p(u)$, $\forall\, u \in X \otimes Y$.

Further (ii) $w^*(u) \leq w(u)$ and $p^*(u) \leq p(u)$.

PROOF : (i) is proved as in prop. 5.4. and (ii) and (iii) follow from definitions.

PROPOSITION 5.6.

Let X, Y, Z be defined as in prop. 5.4. Let X_z denote the set of all bounded linear algebra homomorphisms from X to Z.

Let p^* be defined as in prop. 5.4. then $((X \otimes Y)_z,\ p^*)$ is a l-paranormed algebra.

PROOF : We simply verify that $p^*(uv) \leq p^*(u)\, p^*(v)$, $\forall\, u,\ v \in (X \otimes Y)_z$.

$$\text{Let } u = \sum_{i=1}^{m} x_i \otimes y_i\ ;\ v = \sum_{j=1}^{n} x_j \otimes y_j .$$

$$\text{Then } uv = \sum_{i=1}^{m} \sum_{j=1}^{n} x_i x_j \otimes y_i y_j$$

$$p^*(uv) = \inf\left\{\sum_{i=1}^{m} \sum_{j=1}^{n} \|y_i y_j\| \sup_f \left(p_z f(x_i x_j)\right) ;\ f \in X_z,\ p_{x_z}(f) \leq 1\right)\right\}$$

$$\leq \inf\left\{\sum_{i=1}^{m} \sum_{j=1}^{n} \|y_i\| \|y_j\| \sup_f \left(p_z f(x_i)\, f(x_j)\right) ;\ p_{x_z}(f) \leq 1\right\}$$

(as f is an algebra homomorphism)

$$\leq \inf\left\{\sum_{i=1}^{m} \sum_{j=1}^{n} \|y_i\| \|y_j\| \sup_f p_z f(x_i) \sup_f p_z f(x_j) ;\ p_{x_z}(f) \leq 1\right\}$$

$$= \inf \left\{ \sum_{i=1}^{m} \| y_i \| \sup_f p_z(f(x_i)) \; ; \; p_{x_z}(f) \le 1 \right\} \times \inf \left\{ \sum_{j=1}^{n} \| y_j \| \sup_f p_z f(x_j) \; ; \; p_{x_z}(f) \le 1 \right\}$$

$$= p^*(u) \, p^*(v).$$

PROPOSITION 5.7.

Let X, Y, Z, f be defined as in prop. 5.6. and p be defined as in proposition 5.5.

Then $((X \otimes Y)_z, \, p)$ is a l-paranormed algebra.

PROOF : We verify that $p(uv) \le p(u)\, p(v)$, $\forall$ u, v$\in (X \otimes Y)_z$.

Now it follows from the definition of p(uv) that we are to consider only those $f \in X_z$ for which $p_z f(x_i \, x_j) \le 1$.

As $p_z(f(x_i \, x_j)) \le p_z(f(x_i)) \, p_z(f(x_j))$, so without any loss of generality, we may assume that $p_z(f(x_i)) = 1$, $p_z(f(x_j)) = 1$.

$$\text{Now } p(uv) = \inf \left(\sum_{i,j} \| y_i \, y_j \| \sup_f p_z \left[f \left(\frac{x_i \, x_j}{p_z(f(x_i \, x_j))} \right) ; \; p_z f(x_i \, x_j) \le 1 \, , \; f \ne \theta_{x_z} \right] \right)$$

$$\le \inf \left(\sum_{i,j} \| y_i \, y_j \| \sup_f \left[\left(\frac{1}{p_z(f(x_i \, x_j))} \cdot p_z f(x_i \, x_j) \right) \right] \right) \quad \left(\text{as } 1 \Big/ (p_z f(x_i \, x_j)) \ge 1 \right)$$

$$= \inf \left(\sum_{i,j} \| y_i \, y_j \| \right)$$

$$\le \inf \left(\sum_{i,j} \| y_i \| \, \| y_j \| \right)$$

$$= \inf \left(\sum_{i,j} \| y_i \| \, \| y_j \| \sup_f p_z(f(x_i)) \sup_f p_z(f(x_j)) \; ; \; p_{x_z}(f) \le 1 \right)$$

$$\left[\text{as } \sup_f p_z f(x_i) = 1 = \sup_f p_z(f(x_j)) \right]$$

$$= \inf \left(\sum_i \| y_i \| \sup_f (p_z f(x_i)) \; ; \; p_{x_z}(f) \le 1 \, , \; f \in X_z \right) \times \inf \left(\sum_j \| y_j \| \sup_f p_z f(x_j) \; ; \; p_{x_z}(f) \le 1, \; f \in X_z \right)$$

$$= p^*(u) \, p^*(v) \le p(u) \, p(v).$$

This completes the proof.

5.2. L-TWO PARANORMED ALGEBRA

DEFINITION 5.2.

A l-two paranormed space $(X,\ p,\ p^*)$ over F where X is also an algebra over F is called a l-two paranormed algebra (abbreviated as l-TPA)

if $\forall\ x\ ,\ y \in X\ ,\ p^*(x\,y)\ \le\ p^*(x)\,p^*(y)$ and $p(x\,y)\ \le\ p(x)\,p(y)$.

From propositions 5.4. - 5.7. the following result readily follows :

THEOREM 5.2.

Let X, Z be two l-paranormed algebras over F. Let Y be a normed algebra over F. Then $((X \otimes Y)_z,\ p,\ p^*)$ is a l-two paranormed algebra over F where $p^*,\ p : (X \otimes Y)_z \to R$ are defined respectively as $p^*(u)\ =\ \inf\left(\sum_{i=1}^{n} \|\ y_i\ \|\ \sup_f \left[p_z(f(x_i))\ ,\ p_{x_z}(f)\ \le\ 1\ ,\ f \in X_z\right]\right)$

$$p(u)\ =\ \inf\left(\sum_{i=1}^{n} \|\ y_i\ \|\ \sup_f \left[p_z\left(\frac{f(x_i)}{p(f\ x_i)}\right)\right],\ p_{x_z}(f)\ \le\ 1\ ,\ f \ne \theta_{x_z}\right),$$

infimum being taken over all finite representations of $u\ \in\ (X \otimes Y)_z\ ,\ u\ =\ \sum_{i=1}^{n} x_i \otimes y_i$.

CONCLUDING REMARKS :

1. A linear map between two l-paranormed spaces is bounded if it is continuous. But the converse is not true.

2. Unlike the space of bounded linear maps between two normed linear spaces, the space of bounded linear maps between two l-paranormed spaces is a two normed space only because there exist two distinct nonequivalent norms in the latter case, which, however, coincide in the former case.

3. The presence of two nonequivalent norms ultimately resulted in the introduction of two distinct algebra l-paranorms on the corresponding tensor product of l-paranormed algebras.

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