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Unified Fractional Integral Formulas for the Generalized H–function

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Abstract: The object of this article is to evaluate two unified fractional integrals involving the product of generalized H-function due to Inayat Hussain, Appell function F_3 and a general class of multivariable polynomials due to Srivastava and Garg. These integrals are further applied in proving two theorems on Saigo – Maeda operators of fractional integration. The results obtained provide unification and extension of the results given earlier by Saigo and Raina; Kilbas and Saigo; Saigo and Kilbas; Saxena and Saigo; etc. The results are obtained in a compact form and are useful in preparing the tables of Riemann-Liouville operator, Weyl operator, Erdelyi – Kober operators, Saigo Operator and Saigo and Maeda Operators of fractional calculus.

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1. Introduction and Preliminaries

Fractional calculus deals with the investigations of integrals and derivatives of arbitrary orders. This subject has gained importance during the last three decades or so due to its various applications in many branches of physics and engineering. In this connection, one can refer to [8, 17, 25, 38].

Fractional integrals formulas for the H-function are given by many auhors notably by Raina and Srivastava [18], Srivastava and Hussain [36], Saxena and Nishimoto [29] Saxena and Saigo [31], Kilbas and Saigo[11] and Saigo and Kilbas [22]. In order to provide unification and extension of the aforesaid results on fractional integrals of Special functions scattered in the literature, the authors establish two unified fractional integrals involving the product of generalized H-function [10], Appell function F_3 and a general class of multivariable polynomials [35] . These integrals will be employed in establishing two theorems on Saigo – Maeda operators of fractional integration [23].

Reduction and transformation formulas for hypergeometric series are discussed by Inayat – Hussain [9]. Further in an attempt to derive certain Feynman integrals in two different ways which arise in perturbation calculations of the equilibrium properties of a magnetic model of phase transitions, Inayat – Hussain[10, p. 4126] investigated a generalization of the H-function as

$$\overline{H}(z) = \overline{H}^{m,n}_{p,q}[z] = \overline{H}^{m,n}_{p,q}\left[z \left| \begin{matrix} (\alpha_j, A_j, a_j)_{1,n}, (\alpha_j, A_j)_{n+1,p} \\ (\beta_j, B_j)_{1,m}, (\beta_j, B_j, b_j)_{m+n,q} \end{matrix} \right. \right] \tag{1.1}$$

$$=\frac{1}{2\pi i}\int_L \chi(s) z^s ds, \tag{1.2}$$

$$\text{where } \chi(s) = \frac{\prod_{j=1}^{m}\Gamma(\beta_j - B_j s)\prod_{j=1}^{n}\{\Gamma(1-\alpha_j + A_j s\}^{a_j}}{\prod_{j=m+1}^{q}\{\Gamma(1-\beta_j + B_j s)\}^{b_j}\prod_{j=n+1}^{p}\Gamma(\alpha_j - A_j s)} \tag{1.3}$$

which contains fractional powers of some of the gamma functions $L = L_{i\tau\infty}$ is a contour starting at the point $\tau - i\infty$, terminating at the point $\tau + i\infty$ with $\tau \in R = (-\infty, \infty)$. For a detailed definition, convergence and existence conditions, and for the computable representation of the $\overline{H}$-function the reader is referred to the original papers of Saxena [26], Buschman and Srivastava [2].

It is interesting to note that for $a_i = b_j, \forall i$ and j the $\overline{H}-function$ reduces to the familiar H-function defined by Fox [5], see also Mathai and Saxena [13]. A few interesting special cases of the $\overline{H}$-function [10, pp.4126-4127] which can not be obtained from the H-function are given below.

(I) $g_1 = (-1)^p g(\gamma, \eta, \xi, p; z)$

$$=\frac{K_{d-1}p!\Gamma(1+\beta/2)B(1/2,1/2+\xi/2)}{2^{2+p}\pi\Gamma(\gamma)\Gamma(\gamma-\xi/2)}\left[\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\frac{d\xi(-z)^{\varsigma}\Gamma(-\varsigma)\Gamma(\gamma+\varsigma)\Gamma(\gamma-\xi/2+\varsigma)}{(\eta+\varsigma)^{1+p}\Gamma(1+\xi/2+\varsigma)}\right] \tag{1.4}$$

$$=\frac{K_{d-1}\Gamma(p+1)\Gamma(1/2+\xi/2)}{2^{2+p}\sqrt{\pi}\,\Gamma(\gamma)\Gamma(\gamma-\xi/2)}\times\overline{H}^{1,3}_{3,3}\left[-z\left|\begin{matrix}(1-\gamma,1;1),(1-\gamma+\xi/2,1;1),(1-\eta,1;1+p)\\(0,1),(-\xi/2,1;1),(-\eta,1;1+p)\end{matrix}\right.\right] \tag{1.5}$$

where $K_d \equiv (2^{1-d}\pi^{-d/2}/\Gamma(d/2))$, [10, p.4121, eq. (5)]. The above function is connected with certain class of Feynman integrals.

$$\text{(II) } \beta\,\mathrm{F}(\mathrm{d};\varepsilon) = -\frac{1}{4\pi^{d/2}(1+\varepsilon)^2}\cdot\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\frac{d\varsigma[-(1+\varepsilon)^{-2}]^{\varsigma}\Gamma(-\varsigma)[\Gamma(1+\varsigma)]^2[\Gamma(3/2+\varsigma)]^d}{[\Gamma(2+\varsigma)]^{1+d}} \tag{1.6}$$

$$= -\frac{1}{4\pi^{d/2}(1+\varepsilon)^2}.\overline{H}^{1,2}_{2,2}\left[-(1+\varepsilon)^{-2}\left|\begin{matrix}(0,1;2),(-1/2,1;d)\\(0,1),(-1,1;1+d)\end{matrix}\right.\right] \tag{1.7}$$

$$= -\frac{1}{4\pi^{d/2}(1+\varepsilon)^2}.\overline{H}^{1,3}_{3,2}\left[-(1+\varepsilon)^{-2}\left|\begin{matrix}(0,1;1),(0,1;1),(-1/2,1;d)\\(0,1),(-1,1;1+d)\end{matrix}\right.\right] \tag{1.8}$$

The above function is the exact partition function of the Gaussian model in statistical mechanics.

(III) For further example of a function which is not a special case of the H-function is the poly-logarithm of complex order υ, denoted by $\mathrm{L}^{\upsilon}(z)$. Its relation with $\overline{H}$-function is given by Saxena [26, p.127, eq. (1.12)] as

$$L^{\upsilon}(z) = H\begin{matrix}1,1;\upsilon\\1,2;\upsilon-1\end{matrix}\left[-z\left|\begin{matrix}(1,1;\upsilon)\\(0,1),(0,1;\upsilon-1)\end{matrix}\right.\right] \tag{1.9}$$

An account of $L^{\upsilon}(z)$ is available from the book of Marichev [12].

(IV) Finally the function due to Nagarsenker et al [15,16] also furnishes an example of a function which is not a special case of Fox's H-function.

It has been shown by Buschman and Srivastava [2,p.4708] that the sufficient condition for absolute convergence of the contour integral (1.2) is given by

$$\Omega = \sum_{j=1}^{m}\left|B_j\right| + \sum_{j=1}^{n}\left|a_j A_j\right| - \sum_{j=m+1}^{q}\left|b_j B_j\right| - \sum_{j=n+1}^{p}\left|A_j\right| > 0 \tag{1.10}$$

This condition provides exponential decay of the integrand in (1.2), and region of absolute convergence of (1.2) is given by

$$\left|\arg z\right| < (\pi/2)\Omega \tag{1.11}$$

The asymptotic expansion of $\overline{H}$-function for small and large values of the argument is given below [28]

$$\overline{H}^{m,n}_{p,q}[z] = O\left(|z|^{-\varsigma^*}\right), \text{ for small z, where } \varsigma^* = \underbrace{\min}_{1\le j\le n}\left[\frac{-\operatorname{Re}(\beta_j)}{B_j}\right] \tag{1.12}$$

and $$\overline{H}^{m,n}_{p,q}[z] = O\left(|z|^{-\xi^*}\right), \text{ for large z where } \xi^* = \underbrace{\max}_{1\le j\le n}\left[\frac{-\operatorname{Re}\{a_j(1-\alpha_j)\}}{A_j}\right] \tag{1.13}$$

Abelian theorems and complex inversion theorems for distributional $\overline{H}$-function transformation are established by Saxena and Gupta [27,28]. Functional relations for the $\overline{H}$-function are given by Saxena [26]. Unified fractional integration operators associated with generalized H-function are defined and studied by Saxena and Soni [32]. Recently fractional integral formulas for the $\overline{H}$-function are investigated by Gupta and Soni [6].

A general class of multivariable polynomials is defined and studied by Srivastava and Garg [35] in the following from.

$$S_L^{h_1,\ldots,h_s}(x_1,\ldots,x_s) = \sum_{k_1,\ldots k_s=0}^{h_1k_1+\ldots+h_sk_s\le L}(-L)_{h_1k_1+\ldots+h_sk_s L}A(L;k_1,\cdots,k_s).\frac{x_1^{k_1}}{k_1!}\cdots\frac{x_s^{k_s}}{k_s!},(h_j,\in N; j=1,\ldots,s) \tag{1.14}$$

where $h_1,\ldots,h_s$ are arbitrary positive integers and the coefficients $A(L;k_1,\ldots,k_s),(L,k_j\in N_0; j=1,\ldots\ldots.,s)$ are arbitrary constants, real or complex.

Evidently the case s=1 of the polynomials (1.14) would correspond to the polynomials due to Srivastava [33].

$$S_l^h(x) = \sum_{k=0}^{[1/h]}\frac{(-l)_{hk}}{k!}A_{l,k}x^k \quad , \qquad \left(l\in N_0 = \{0,1,2,\ldots\}\right) \tag{1.15}$$

where $(\lambda)_j = \Gamma(\lambda + j)/\Gamma(\lambda)$, h is an arbitrary positive integer and the coefficients $A_{l,k}(l,k \in N_0)$ are arbitrary constants, real or complex.

Integral operators involving the polynomial (1.14) are defined and studied by the first two authors [30]. Some multidimensional fractional integral operators involving the polynomials $S_L^{h_1,\dots,h_s}(.)$ are defined and studied by Srivastava, Saxena and Ram [39].

Now we recall here the definition of the following generalized fractional integration operators of arbitrary order involving Appell function F_3 [23, p.393, eq.(4.12) and (4.13)] in the kernel introduced and studied by Saigo and Maeda in the following form.

Let $a, a', b, b', c \in C$ and $x > 0$, then

$$\left(I_{0+}^{a,a',b,b',c} f\right)(x) = \frac{x^{-a}}{\Gamma(c)} \int_0^x (x-t)^{c-1} t^{-a'} F_3(a,a',b,b';c;1-t/x,1-x/t) f(t)dt, (\mathrm{Re}(c) > 0) \quad (1.16)$$

$$\left(I_{0+}^{a,a',b,b',c} f\right)(x) = \left(\frac{d}{dx}\right)^k \left(I_{0+}^{a,a',b+k,b',c+k} f\right)(x), \left(\mathrm{Re}(c) \le 0; k = [-\mathrm{Re}(c)] + 1\right) \quad (1.17)$$

$$\left(I_{-}^{a,a',b,b',c} f\right)(x) = \frac{x^{-a'}}{\Gamma(c)} \int_x^\infty (t-x)^{c-1} t^{-a} F_3(a,a',b,b';c;1-x/t,1-t/x) f(t)dt, (\mathrm{Re}(c) > 0) \quad (1.18)$$

$$\left(I_{-}^{a,a',b,b',c} f\right)(x) = \left(-\frac{d}{dx}\right)^k \left(I_{-}^{a,a',b,b'+k,c+k} f\right)(x), \left(\mathrm{Re}(c) \le 0; k = [-\mathrm{Re}(c)] + 1\right) \quad (1.19)$$

The following results are required in the proofs : [25, p.727, Eq.(5.4.51.2)].

$$\int_0^x t^{\rho-1}(x-t)^{c-1} F_3(a,a',b,b';c;1-t/x,1-x/t)dt = \Gamma\left[\begin{matrix} c, \rho+a', \rho+b', \rho+c-a-b \\ \rho+a'+b', \rho+c-a, \rho+c-b \end{matrix}\right] x^{\rho+c-1} \quad (1.20)$$

where $\mathrm{Re}(c) > 0; \mathrm{Re}(\rho) > \max[\mathrm{Re}(-a'), \mathrm{Re}(-b'), \mathrm{Re}(a+b-c)]$ and

$$\int_x^\infty t^{\rho-1}(t-x)^{c-1} F_3(a,a',b,b';c;1-x/t,1-t/x)dt = \Gamma\left[\begin{matrix} c, 1+a'-c-\rho, 1+b'-c-\rho, 1-a-b-\rho \\ 1-\rho+a'+b'-c, 1-\rho-a, 1-\rho-b \end{matrix}\right] x^{\rho+c-1} \quad (1.21)$$

where $\mathrm{Re}(c) > 0; \mathrm{Re}(\rho) < 1 + \min[\mathrm{Re}(a'-c), \mathrm{Re}(b'-c), \mathrm{Re}(-a-b)]$.

The symbol $\Gamma\left[\begin{matrix} \cdots \\ \cdots \end{matrix}\right]$ occurring in (1.20) and (1.21) represents the quotient of the product of gamma functions.

2. Integrals

The integral to be established here is

$$\int_0^x t^{\rho-1}(x-t)^{c-1} F_3(a,a',b,b';c;1-t/x,1-x/t) S_L^{h_1,\dots,h_s}(y_1 t^{\lambda_1}, \cdots, y_s t^{\lambda_s}) \overline{H}_{p,q}^{m,n}(\lambda t^\sigma)dt$$

$$= x^{\rho+c-1} \sum_{k_1,\dots,k_s=0}^{h_1k_1+\dots+h_sk_s \le L} (-L)_{h_1k_1+\dots+h_sk_s} A(L;k_1,\cdots,k_s). \frac{y_1^{k_1}}{k_1!} \cdots \frac{y_s^{k_s}}{k_s!} x^{k_1\lambda_1+\dots+k_s\lambda_s} \Gamma(c)$$

$$\times \overline{H}{}^{m,n+3}_{p+3,q+3}\left[(\lambda x^{\sigma})\middle|\begin{matrix}(\theta-a',\sigma),(\theta-b',\sigma),(\theta+a+b-c,\sigma),(\alpha_j,A_j,a_j)_{1,n},(\alpha_j,A_j)_{n+1,p}\\(\beta_j,B_j)_{1,m},(\beta_j,B_j,b_j)_{m+1,q},(\theta-a'-b',\sigma),(\theta+a-c,\sigma),(\theta+b-c,\sigma)\end{matrix}\right] \quad (2.1)$$

where $\mathrm{Re}(c)>0;\theta=1-\rho-\sum_{j=1}^{s}\lambda_j k_j,\sigma>0,\lambda_j>0,(j=1,\cdots,s),|\arg\lambda|<(\pi/2)\,\Omega,\Omega>0$

$$\sigma\max[\tau,\varsigma^{*}]<\mathrm{Re}(\rho)+\min[\mathrm{Re}(a'),\mathrm{Re}(b'),\mathrm{Re}(c-a-b)] \quad (2.2)$$

Proof : To establish the integral (2.1), we express the $\overline{H}$-function in terms of Mellin-Barnes contour integral given by (1.2), general class of multivariable polynomials in the series form from (1.14) and then interchanging the order of summation and integration and the order of t and ξ - integrals, which is justified under the conditions stated with (2.1), so that the integral (say Δ) transforms into the form

$$\Delta=\sum_{k_1,\dots,k_s=0}^{h_1k_1+\dots+h_sk_s\le L}(-L)_{h_1k_1+\dots+h_sk_s}A(L;k_1,\cdots,k_s).\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}\times$$

$$\frac{1}{2\pi i}\int_L\chi(\xi)\lambda^{\xi}\int_0^x t^{\rho+\sum_{j=1}^{s}\lambda_jk_j+\sigma\xi-1}(x-t)^{c-1}F_3(a,a',b,b';c;1-t/x,1-x/t)dt\,d\xi$$

On evaluating the t-integral by means of (1.20), we find that

$$\Delta=x^{\rho+c-1}\sum_{k_1,\dots,k_s=0}^{h_1k_1+\dots+h_sk_s\le L}(-L)_{h_1k_1+\dots+h_sk_s}A(L;k_1,\cdots,k_s).\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}x^{k_1\lambda_1+\dots+k_s\lambda_s}\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\chi(\xi)(\lambda x^{\sigma})^{\xi}$$

$$\times\Gamma\left[\begin{matrix}c,\rho+a'+\sum_{j=1}^{s}\lambda_jk_j+\sigma\xi,\rho+b'+\sum_{j=1}^{s}\lambda_jk_j+\sigma\xi,\rho+\sum_{j=1}^{s}\lambda_jk_j+c-a-b+\sigma\xi\\\rho+a'+b'+\sum_{j=1}^{s}\lambda_jk_j+\sigma\xi,\rho+\sum_{j=1}^{s}\lambda_jk_j+\sigma\xi+c-a,\rho+\sum_{j=1}^{s}\lambda_jk_j+\sigma\xi+c-b\end{matrix}\right]d\xi$$

On interpreting the above contour integral in terms of $\overline{H}$-function, the desired result (2.1) follows.

The following integral can be proved in the same way, if we use (2.2) instead of (2.1)

$$\int_x^{\infty}t^{\rho-1}(t-x)^{c-1}F_3(a,a',b,b';c;1-x/t,1-t/x)S_L^{h_1,\dots,h_s}(y_1t^{\lambda_1},\cdots,y_st^{\lambda_s})\overline{H}{}^{m,n}_{p,q}(\lambda t^{\sigma})dt$$

$$=x^{\rho+c-1}\sum_{k_1,\dots,k_s=0}^{h_1k_1+\dots+h_sk_s\le L}(-L)_{h_1k_1+\dots+h_sk_s\le L}A(L;k_1,\cdots,k_s).\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}x^{k_1\lambda_1+\dots+k_s\lambda_s}\Gamma(c)$$

$$\times \overline{H}{}^{m,n+3}_{p+3,q+3}\left[(\lambda x^{\sigma})\middle|\begin{matrix}(\theta-a,\sigma),(\theta-b,\sigma),(\theta+a'+b'-c,\sigma),(\alpha_j,A_j,a_j)_{1,n},(\alpha_j,A_j)_{n+1,p}\\(\beta_j,B_j)_{1,m},(\beta_j,B_j,b_j)_{m+1,q},(\theta+a'-c,\sigma),(\theta+b'-c,\sigma),(\theta-a-b,\sigma)\end{matrix}\right] \quad (2.3)$$

where $\mathrm{Re}(c)>0;\theta=1-\rho-\sum_{j=1}^{s}\lambda_jk_j,\sigma>0,\lambda_j>0,(j=1,\cdots,s),|\arg\lambda|<(\pi/2)\Omega,\Omega>0$

$$\sigma\min[\tau,\xi^{*}]+1>\mathrm{Re}(\rho)+\max[\mathrm{Re}(\gamma-a'),\mathrm{Re}(\gamma-\beta'),\mathrm{Re}(\alpha+\beta-\alpha')] \quad (2.4)$$

In what follows θ stand for $1-\rho-\sum_{j=1}^{s}\lambda_j k_j$ (2.5)

3. Applications

As an application of the results derived in the preceding section we establish two theorems which provide expression for the generalized fractional integration of $\overline{H}$-function associated with a general class of multivariable polynomials under Saigo-Maeda operators.

Theorem 1: Let $a,a',b,b',c\in C,\mathrm{Re}(c)>0,\sigma>0$. Further let the constants $m,n,p,q\in N_0$; $A_i,B_j\in R_+$; $\alpha_i,\beta_j\in R$ or C $(i=1,\cdots,p;j=1,\cdots,q)$ and the exponents $a_i(i=1,\cdots,n)$ and $b_j(j=m+1,\cdots,q)\notin N$; $|\arg\lambda|<(\pi/2)\Omega,\Omega>0$, satisfy the conditions

$$\sigma\max[\tau,\varsigma^*]<\mathrm{Re}(\rho)+\min[0,\mathrm{Re}(b'-a'),\mathrm{Re}(c-a-b-b')],|\arg\lambda|<(\pi/2)\,\Omega,\Omega>0. \quad (3.1)$$

Then the generalized fractional integral $I_{0+}^{a,a',b,b',c}$ of the product $\overline{H}$-function and $S_L^{k_1,\ldots,k_x}(.)$ exists and there holds the formula

$$\left(I_{0+}^{a,a',b,b',c}[t^{\rho-1}\overline{H}\left(\lambda t^{\sigma}\right)S_L^{h_1,\cdots,h_s}(y_1t^{\lambda_1},\cdots,y_st^{\lambda_s})]\right)(x)$$

$$=x^{\rho+c-a-a'-1}\sum_{k_1,\ldots,k_s=0}^{h_1k_1+\ldots+h_sk_s}(-L)_{h_1k_1+\ldots+h_sk_s}A(L;k_1,\cdots,k_s).\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}x^{k_1\lambda_1+\ldots+k_s\lambda_s}$$

$$\times H_{p+3,q+3}^{m,n+3}\left[(\lambda x^{\sigma})\middle|\begin{matrix}(\alpha_j,A_j,a_j)_{1,n},(\theta+a+a'+b-c,\sigma),(\theta,\sigma),(\theta+a'-b',\sigma),(\alpha_j,A_j)_{n+1,p}\\(\beta_j,B_j)_{1,m},(\beta_j,B_j,b_j)_{m+1,q},(\theta+a+a'-c,\sigma),(\theta+a'+b-c,\sigma),(\theta-b',\sigma)\end{matrix}\right] \quad (3.2)$$

For $a_i=b_j=1,\forall$ i and j (3.2) reduces to

Corollary 1.1: Let $a,a',b,b',c\in C,\mathrm{Re}(c)>0,\sigma>0$. Further let the constants $m,n,p,q\in N_0$; $A_i,B_j\in R_+$; $\alpha_i,\beta_j\in R$ or C $(i=1,\cdots,p;j=1,\cdots,q)$, $|\arg\lambda|<(\pi/2)\Omega^*,\Omega^*>0$, satisfy the condition

$$\sigma\max[\tau,\varsigma^*]<\mathrm{Re}(\rho)+\min[0,\mathrm{Re}(b'-a'),\mathrm{Re}(c-a-b-a')] \quad (3.3)$$

then the following result holds :

$$\left(I_{0+}^{a,a',b,b',c}[x^{\rho-1}H\left(\lambda x^{\sigma}\right)S_L^{h_1,\cdots,h_s}(y_1t^{\lambda_1},\cdots,y_st^{\lambda_s})]\right)(x)$$

$$=x^{\rho+c-a-a'-1}\sum_{k_1,\ldots,k_s=0}^{h_1k_1+\ldots+h_sk_s\le L}(-L)_{h_1k_1+\ldots+h_sk_s}A(L;k_1,\cdots,k_s).\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}x^{k_1\lambda_1+\ldots+k_s\lambda_s}$$

$$\times\overline{H}_{p+3,q+3}^{m,n+3}\left[(\lambda x^{\sigma})\middle|\begin{matrix}(\alpha_j,A_j)_{1,n},(\theta+a+a'+b'-c,\sigma),(\theta,\sigma),(\theta+a'-b',\sigma),(\alpha_j,A_j)_{n+1,p}\\(\beta_j,B_j)_{1,m},(\beta_j,B_j)_{m+1,q},(\theta+a+a'-c,\sigma),(\theta+a+b'-c,\sigma),(\theta-b,\sigma)\end{matrix}\right] \quad (3.4)$$

where H(.) represents the H-function of Fox [5] (also see [14] and $|\arg\lambda|>(\pi/2)\Omega^*$ and

$$\Omega^*=\sum_{j=1}^{n}|A_j|-\sum_{j=n+1}^{p}|A_j|+\sum_{j=1}^{m}|B_j|-\sum_{j=m+1}^{q}|B_j|>0.$$

If we set $a'=0$, then by virtue of the identity [31, p.93, eq.2.15]

$$\left(I_{0+}^{\alpha,0,\beta,\beta',\gamma} f\right)(x) = \left(I_{0+}^{\gamma;\alpha-\gamma,-\beta} f\right)(x), \quad (\gamma \in C) \tag{3.5}$$

where the operator on the right is due to Saigo [20], we arrive at

Corollary 1.2: Let $\alpha,\beta,\eta,\sigma \in C, \mathrm{Re}(\alpha)>0, \sigma>0$. Further let the constants $m,n,p,q \in N_0$; $A_i, B_j \in R_+$; $\alpha_i, \beta_j \in R$ or C $(i=1,\cdots,p; j=1,\cdots,q)$, and the exponents $a_i (i=1,\cdots,n)$ and $b_j (j=m+1,\cdots,q) \notin N$; $|\arg\lambda| < (\pi/2)\Omega$, $\Omega>0$, satisfy the condition

$$\sigma \max[\tau, \varsigma^*] < \mathrm{Re}(\rho) + \min[0, \mathrm{Re}(\eta-\beta)] > 0 \tag{3.6}$$

$$\left(I_{0+}^{\alpha,\beta,\eta}[x^{\rho-1}\overline{H}\left(\lambda x^{\sigma}\right)S_L^{h_1,\cdots,h_s}(y_1 t^{\lambda_1},\cdots,y_s t^{\lambda_s})]\right)(x)$$

$$= x^{\rho-\beta-1} \sum_{k_1,\cdots,k_s=0}^{h_1k_1+\ldots+h_sk_s \le L} (-L)_{h_1k_1+\ldots+h_sk_s} A(L;k_1,\cdots,k_s).\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!} x^{k_1\lambda_1+\ldots+k_s\lambda_s}$$

$$\times \overline{H}\,{}^{m,n+2}_{p+2,q+2}\left[(\lambda x^{\sigma})\left| \begin{matrix} (\alpha_j,A_j,a_j)_{1,n},(\theta,\sigma),(\theta+\alpha+\beta+\eta,\sigma),(\alpha_j,A_j)_{n+1,p} \\ (\beta_j,B_j)_{1,m},(\beta_j,B_j,b_j)_{m+1,q},(\theta+\beta,\sigma),(\theta+\eta,\sigma) \end{matrix}\right.\right] \tag{3.7}$$

If we set $a_i = b_j = 1, \forall\ i$ and j, $h_j = 0\ (j=2,\cdots,s)$ and $L \to 0$, we arrive at a known result given by Saigo and Kilbas [22, p.35, Theorem 1]. On the other hand if $a_i = b_j = 1, \forall\ i$ and j, then using the identity

$$H\,{}^{1,0}_{0,1}\left[x\left| \begin{matrix} - \\ (0,1) \end{matrix}\right.\right] = e^{-x} \tag{3.8}$$

and setting $h_j = 0\ (j=2,\cdots,s)$ and $L \to 0$ in (3.7), we then arrive at another known result [24, p.17, Lemma 4].

If, however, we take $\beta = -\alpha$ in (3.7), then we obtain the following interesting result for Riemann-Liouville operator defined by

$$\left(I_{0+}^{\alpha} f\right)(x) = \left(1/\Gamma(\alpha)\right)\int_0^x (x-t)^{\alpha-1} f(t)\,dt \quad , \tag{3.9}$$

in the form $\left(I_{0+}^{\alpha}[t^{\rho-1}\overline{H}\left(\lambda t^{\sigma}\right)S_L^{h_1,\cdots,h_s}(y_1 t^{\lambda_1},\cdots,y_s t^{\lambda_s})]\right)(x)$

$$= x^{\rho-\alpha-1} \sum_{k_1,\cdots,k_s=0}^{h_1k_1+\ldots+h_sk_s \le L} (-L)_{h_1k_1+\ldots+h_sk_s} A(L;k_1,\cdots,k_s).\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!} x^{k_1\lambda_1+\ldots+k_s\lambda_s}$$

$$\times \overline{H}\,{}^{m,n+1}_{p+1,q+1}\left[(\lambda x^{\sigma})\left| \begin{matrix} (\alpha_j,A_j,a_j)_{1,n},(\theta,\sigma),(\alpha_j,A_j)_{n+1,p} \\ (\beta_j,B_j)_{1,m},(\beta_j,B_j,b_j)_{m+1,q},(\theta-\alpha,\sigma) \end{matrix}\right.\right] \tag{3.10}$$

which holds under the conditions given in (3.7) with $\beta = -\alpha$.

Note : For $\beta = 0$ (3.7) gives the results for Erdelyi-Kober Operators.

Next, if we set $A(L;k_1,\cdots,k_s) = \psi$, where

$$\psi = \frac{\prod_{j=1}^{E}(e_j)_{k_1\varsigma_j^{(1)}+\cdots+k_s\varsigma_j^{(s)}}\prod_{j=1}^{U^{(1)}}(u_j^{(1)})_{k_1x_j^{(1)}}\cdots\prod_{j=1}^{U^{(s)}}(u_j^{(s)})_{k_xx_j^{(s)}}}{\prod_{j=1}^{G}(g_j)_{k_1\tau_j^{(1)}+\cdots+k_x\tau_j^{(s)}}\prod_{j=1}^{V^{(1)}}(v_j^{(1)})_{k_1\xi_j^{(1)}}\cdots\prod_{j=1}^{V^{(s)}}(v_j^{(s)})_{k_x\xi_j^{(s)}}} \quad (3.11)$$

then $S_L^{m_1,\cdots,m_s}[y_1,\cdots,y_s]$ reduces to

$$S_L^{m_1,\cdots,m_s}[y_1,\cdots,y_s]=F^{1+E:U^{(1)};\cdots;U^{(s)}}_{G:V^{(1)};\cdots;V^{(s)}}\left[\begin{matrix}[-L:m_1,\cdots,m_s],[e:\varsigma^{(1)},\cdots,\varsigma^{(x)}];[(u^{(1)}):w^{(1)}];\cdots;[(u^{(s)}):w^{(s)}];y_1\\ [g:\tau^{(1)},\cdots,\tau^{(s)}];\ [v^{(1)}:\xi^{(1)}];\cdots;[(v^{(s)}):\xi^{(s)}];y_s\end{matrix}\right] \quad (3.12)$$

and consequently we arrive at

Corollary 1.3: Let $a,a',b,b',c \in C, \mathrm{Re}(c)>0, \sigma>0$. Further let the constants $m,n,p,q \in N_0;\ A_i,B_j \in R;\ \alpha_i,\beta_j \in R$ or $C\ (i=1,\cdots,p;\ j=1,\cdots,q)$, $|\arg\lambda|<(\pi/2)\Omega, \Omega>0$, satisfy the conditions

$$\sigma\max[\tau,\varsigma^*] < \mathrm{Re}(\rho)+\min[0,\mathrm{Re}(b'-a'),\mathrm{Re}(c-a-b-a')] \quad (3.13)$$

Then the generalized fractional integral of $I_{0+}^{a,a',b,b',c}$ of the product of generalized H-function and generalized Lauricella $F^{1+E:U^{(1)};\cdots;U^{(s)}}_{G:V^{(1)};\cdots;V^{(x)}}$ exists and there holds the formula

$$\left(I_{0+}^{a,a',b,b',c}\left[t^{\rho-1}\overline{H}(\lambda t^{\sigma})F^{1+E:U^{(1)};\cdots;U^{(s)}}_{G:V^{(1)};\cdots;V^{(s)}}\left[\begin{matrix}[-L:m_1,\cdots,m_s],[e:\varsigma^{(1)},\cdots,\varsigma^{(s)}];[(u^{(1)}):w^{(1)}];\cdots;[(u^{(s)}):w^{(s)}];y_1t^{\lambda_1}\\ [g:\tau^{(1)},\cdots,\tau^{(s)}];\ [v^{(1)}:\xi^{(1)}];\cdots;[(v^{(s)}):\xi^{(s)}];y_st^{\lambda_s}\end{matrix}\right]\right]\right)(x)$$

$$= x^{\rho+c-a-a'-1}\sum_{k_1,\ldots,k_s=0}^{h_1k_1+\ldots+h_sk_s\le L}(-L)_{h_1k_1+\ldots+h_sk_s}\,\psi\,.\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}x^{k_1\lambda_1+\ldots+k_s\lambda_s}\times$$

$$\overline{H}^{m,n+3}_{p+3,q+3}\left[(\lambda x^{\sigma})\middle|\begin{matrix}(\alpha_j,A_j,a_j)_{1,n},(\theta+a+a'+b'-c,\sigma),(\theta,\sigma),(\theta+a-b,\sigma),(\alpha_j,A_j)_{n+1,p}\\(\beta_j,B_j)_{1,m},(\beta_j,B_j,b_j)_{m+1,q},(\theta+a+a'-c,\sigma),(\theta+a+b'-c,\sigma),(\theta-b,\sigma)\end{matrix}\right] \quad (3.14)$$

Theorem 2: Let $a,a',b,b',c \in C, \mathrm{Re}(c)>0, \sigma>0$. Further let the constants $m,n,p,q \in N_0;\ A_i,B_j \in R_+\ (i=1,\cdots,p;\ j=1,\cdots,q);\ \alpha_i,\beta_j \in R$ or $C(i=1,\cdots,p;\ j=1,\cdots,q)$ and the exponents $a_i,b_j (i=1,\cdots,n, j=m+1,\cdots,q) \notin N$; $|\arg\lambda|<(1/2)\pi\Omega, \Omega>0$, satisfy the conditions

$$\sigma\min[\tau,\xi^*]+1 > \mathrm{Re}(\rho)+\max[\mathrm{Re}(c-a-a'),\mathrm{Re}(c-a-b'),\mathrm{Re}(b)] \quad (3.15)$$

Then the generalized fractional integral $I_-^{a,a',b,b',c}$ of the product $\overline{H}$-function and $S_L^{m_1,\ldots,m_s}(.)$ exists and the following relation holds:

$$\left(I_-^{a,a',b,b',c}[t^{\rho-1}\overline{H}(\lambda t^{\sigma})S_L^{h_1,\cdots,h_s}(y_1t^{\lambda_1},\cdots,y_st^{\lambda_s})]\right)(x)$$

$$= x^{\rho+c-a-a'-1}\sum_{k_1,\ldots,k_s=0}^{h_1k_1+\ldots+h_sk_s\le L}(-L)_{h_1k_1+\ldots+h_sk_s}A(L;k_1,\cdots,k_s).\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}x^{k_1\lambda_1+\ldots+k_s\lambda_s}$$

$$\overline{H}\ {}^{m+3,n}_{p+3,q+3}\left[(\lambda x^{\sigma})\middle|\begin{matrix}(\alpha_j,A_j,a_j)_{1,n},(\alpha_j,A_j)_{n+1,p},(\theta+a+a'+b-c,\sigma),(\theta,\sigma),(\theta+a'-b,\sigma),\\(\theta+a+a'-c,\sigma),(\theta+a+b'-c,\sigma),(\theta-b,\sigma)(\beta_j,B_j)_{1,m},(\beta_j,B_j)_{m+1,q},\end{matrix}\right] \quad (3.16)$$

When $a_i = b_j = 1, \forall\ i$ and j. we arrive at

Corollary 2.1: Let $a,a',b,b',c \in C, \mathrm{Re}(c) > 0, \sigma > 0$. Further let the constants $m,n,p,q \in N_0;\ A_i,B_j \in R_+\ (i=1,\cdots,p; j=1,\cdots,q);\ \alpha_i,\beta_j \in R$ or $\mathrm{C}(i=1,\cdots,p; j=1,\cdots,q)$, $|\arg\lambda| < (1/2)\pi\Omega^*, \Omega^* > 0$, satisfy the condition

$$\sigma \min[\tau,\xi^*] + 1 > \mathrm{Re}(\rho) + \max[\mathrm{Re}(c-a-a'), \mathrm{Re}(c-a-b'), \mathrm{Re}(b)] \quad (3.17)$$

then the generalized fractional integral $I_-^{a,a',b,b',c}$ of the product of the H-function and $S_L^{h_1,\dots,h_s}(.)$ exists and the following relation holds :

$$\left(I_-^{a,a',b,b',c}\left[t^{\rho-1}H\left(\lambda t^{\sigma}\right)S_L^{h_1,\dots,h_s}(y_1 t^{\lambda_1},\cdots,y_s t^{\lambda_s})\right]\right)(x)$$

$$= x^{\rho+c-a-a'-1}\sum_{k_1,\dots,k_s=0}^{h_1k_1+\dots+h_sk_s\le L}(-L)_{h_1k_1+\dots+h_sk_s}A(L;k_1,\cdots,k_s).\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}x^{k_1\lambda_1+\dots+k_s\lambda_s}$$

$$\times H\ {}^{m+3,n}_{p+3,q+3}\left[(\lambda x^{\sigma})\middle|\begin{matrix}(\alpha_j,A)_{1,p},(\theta+\alpha+a'+b'-c,\sigma),(\theta,\sigma),(\theta+a-b,\sigma)\\(\theta+a+a'-c,\sigma),(\theta+a+b'-c,\sigma),(\theta-b,\sigma),(\beta_j,B_j)_{1,q}\end{matrix}\right] \quad (3.18)$$

where Ω^* is defined in equation (3.4) and ς^* in equation (1.13).

Corollary 2.2: Let $a,a',b,b',c \in C, \mathrm{Re}(c) > 0, \sigma > 0$. Further let the constants $m,n,p,q \in N_0;\ A_i,B_j \in R_+\ (i=1,\cdots,p; j=1,\cdots,q);\ \alpha_i,\beta_j \in R$ or $\mathrm{C}(i=1,\cdots,p; j=1,\cdots,q)$ and the exponents $a_i,b_j (i=1,\cdots,n, j=m+1,\cdots,q) \notin N;\ |\arg\lambda| < (1/2)\pi\Omega, \Omega > 0$, satisfy the condition

$$\sigma\min[\tau,\varsigma^*]+1 > \mathrm{Re}(\rho)+\max[\mathrm{Re}(-\beta),\mathrm{Re}(-\eta)] \quad (3.19)$$

Then the generalized fractional integral $I_-^{\alpha,\beta,\eta}$ of the product $\overline{H}$-function and $S_L^{h_1,\dots,h_x}(.)$ exists and the following relation holds:

$$\left(I_-^{\alpha,\beta,\eta}\left[t^{\rho-1}\overline{H}\left(\lambda t^{\sigma}\right)S_L^{h_1,\cdots,h_s}(y_1t^{\lambda_1},\cdots,y_st^{\lambda_s})\right]\right)(x)$$

$$= x^{\rho-\beta-1}\sum_{k_1,\dots,k_s=0}^{h_1k_1+\dots+h_sk_s\le L}(-L)_{h_1k_1+\dots+h_sk_s}A(L;k_1,\cdots,k_s).\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}x^{k_1\lambda_1+\dots+k_s\lambda_s}$$

$$\times\overline{H}\ {}^{m+2,n}_{p+2,q+2}\left[(\lambda x^{\sigma})\middle|\begin{matrix}(\alpha_j,A_j,a_j)_{1,n},(\alpha_j,A_j)_{n+1,p},(\theta,\sigma),(\theta+\alpha+\beta+\eta,\sigma)\\(\theta+\beta,\sigma),(\theta+\eta,\sigma),(\beta_j,B_j)_{1,m},(\beta_j,B_j,b_j)_{m+1,q}\end{matrix}\right] \quad (3.20)$$

If we set $a_i = b_j = 1, \forall\ i$ and j, $h_j = 0\ (j=2,\cdots,s)$ and $L \to 0$, we obtain a known result given earlier by Saigo and Kilbas [22, p.38, Theorem 2].

On the other hand if we take $m=q=1, n=p=0$, use the identity (3.8), then on setting $h_j = 0\ (j=2,\cdots,s)$ we obtain another known result [24, p.17, Lemma 4].

Further, if we take $\beta = -\alpha$ in (3.20), then the following results holds:

$$\left(I_{-}^{\alpha}\left[t^{\rho-1}\overline{H}_{p,q}^{m,n}\left(\lambda t^{\sigma}\right)S_{L}^{h_1,\cdots,h_s}(y_1 t^{\lambda_1},\cdots,y_s t^{\lambda_s})\right]\right)(x)$$

$$= x^{\rho-\alpha-1}\sum_{k_1,\ldots,k_s=0}^{h_1k_1+\ldots+h_sk_s\le L}(-L)_{h_1k_1+\ldots+h_sk_s}A(L;k_1,\cdots,k_s).\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}x^{k_1\lambda_1+\ldots+k_s\lambda_s}$$

$$\times\overline{H}_{p+1,q+1}^{m+1,n}\left[(\lambda x^{\sigma})\middle|\begin{matrix}(\alpha_j,A_j,a_j)_{1,n},(\theta,\sigma),(\alpha_j,A_j)_{n+1,p}\\(\beta_j,B_j)_{1,m},(\beta_j,B_j,b_j)_{m+1,q},(\theta-\alpha,\sigma)\end{matrix}\right] \qquad (3.21)$$

which holds under the same conditions given in (3.20) with $\beta = -\alpha$.

and where $\qquad \left(I_{-}^{\alpha}f\right)(x) = \left(1/\Gamma(\alpha)\right)\int_{x}^{\infty}(t-x)^{\alpha-1}f(t)dt, \quad \left(\mathrm{Re}(\alpha)\right)>0 \qquad (3.22)$

If we set $A(L;k_1,\cdots,k_s)=\phi$, where

$$\phi = \frac{\prod_{j=1}^{E}(e_j)_{k_1\varsigma_j^{(1)}+\cdots+k_s\varsigma_j^{(s)}}\prod_{j=1}^{U^{(1)}}(u_j^{(1)})_{k_1x_j^{(1)}}\cdots\prod_{j=1}^{U^{(s)}}(u_j^{(s)})_{k_sx_j^{(s)}}}{\prod_{j=1}^{G}(g_j)_{k_1\tau_j^{(1)}+\cdots+k_s\tau_j^{(s)}}\prod_{j=1}^{V^{(1)}}(v_j^{(1)})_{k_1\xi_j^{(1)}}\cdots\prod_{j=1}^{V^{(s)}}(v_j^{(s)})_{k_s\xi_j^{(s)}}}$$

then $S_{L}^{m_1,\cdots,m_s}[y_1,\cdots,y_s]$ reduces to a generalized Lauricella function of Srivastava and Daoust [34, p.454] and we arrive at

Corollary 2.3: Let $a,a',b,b',c \in C, \mathrm{Re}(c)>0, \sigma>0$. Further let the constants $m,n,p,q \in N_0$; $A_i, B_j \in R$; $\alpha_i,\beta_j \in R$ or C $(i=1,\cdots,p; j=1,\cdots,q)$, $|\arg\lambda|<(\pi/2)\Omega, \Omega>0$, satisfy the conditions

$$\sigma\min[\tau,\xi^*]+1>\mathrm{Re}(\rho)+\max[\mathrm{Re}(c-a-a'),\mathrm{Re}(c-a-b),\mathrm{Re}(b)] \qquad (3.23)$$

Under above mentioned conditions the generalized fractional Integral $I_{-}^{a,a',b,b',c}$ of the product of $\overline{H}(\lambda t^{\sigma})$ and generalized Lauricella function $F_{G:V^{(1)};\cdots;V^{(s)}}^{1+E:U^{(1)};\cdots;U^{(s)}}$ exists and there holds the formula :

$$\left(I_{-}^{a,a',b,b',c}\left[t^{\rho-1}\overline{H}(\lambda t^{\sigma})F_{G:V^{(1)};\cdots;V^{(s)}}^{1+E:U^{(1)};\cdots;U^{(s)}}\left[\begin{matrix}[-L:m_1,\cdots,m_s],[e:\varsigma^{(1)},\cdots,\varsigma^{(s)}];[(u^1):w^{(1)}];\cdots;[(u^{(s)}):w^{(s)}];y_1t^{\lambda_1}\\ [g:\tau^{(1)},\cdots,\tau^{(s)}];[v^{(1)}:\xi^{(1)}];\cdots;[(v^{(s)}):\xi^{(s)}];y_st^{\lambda_s}\end{matrix}\right]\right]\right)(x)$$

$$= x^{\rho+c-a-a'-1}\sum_{k_1,\ldots,k_s=0}^{h_1k_1+\ldots+h_sk_s\le L}(-L)_{h_1k_1+\ldots+h_sk_s}\phi.\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}x^{k_1\lambda_1+\ldots+k_s\lambda_s}\times$$

$$\overline{H}_{p+3,q+3}^{m+3,n}\left[(\lambda x^{\sigma})\middle|\begin{matrix}(\alpha_j,A_j,a_j)_{1,n},(\alpha_j,A_j)_{n+1,p},(\theta+a+a'+b'-c,\sigma),(\theta,\sigma),(\theta+a-b,\sigma)\\(\theta+a+a'-c,\sigma),(\theta+a+b'-c,\sigma),(\theta-b,\sigma),(\beta_j,B_j)_{1,m},(\beta_j,B_j,b_j)_{m+1,q},\end{matrix}\right] \qquad (3.24)$$

Finally, it is interesting to observe that due to fairly general character $\overline{H}$ -function and generalized Lauricella function, numerous interesting special cases of (3.2) and (3.16) associated with special functions of one variable, orthogonal and non-orthogonal polynomials including Badient's polynomials, Sister Ceiline's polynomials and Rice's polynomials etc. can be derived by specializing the parameters suitably but for lack of space such results are not reported here.

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